On Some Crack Problems in Generalized Thermoelasticity

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Introduction
INTRODUCTION

In recent years, considerable effort has been devoted to the study of cracks in solids. This is due to their applications in industry in general and the fabrication of electronic components in particular. Most of the studies of dynamical crack problems are done using the equations of the coupled or uncoupled theories of thermoelasticity. This is suitable for most situations where long time effects are sought. However, when short time effects are important, as in many practical situations, then the full system of generalized thermoelastic equations must be used.

Failure of structures frequently occurs because of cracks that extend beyond a safe size. All structures contain cracks, as manufacturing defects or as a result of service loading, which can be either mechanical or thermal. If the load is applied cyclically, then the crack may grow in fatigue to final fracture. As the size of the crack increases, the residual strength of the structure decreases. In the final stages of the crack growth, the rate increases suddenly leading to catastrophic structural failure.

Study of fracture mechanics helps to maintain the structural integrity due to cracks. According to a report in the United States [1], the costs of fracture was estimated in 1982 to be $119 billion per year, which was equivalent to 4% of gross national product. The same report indicated that an estimated $35 billion dollars could be saved through the current available fracture technology. A similar study
commissioned by the European Union estimated that 80 billion ECU per year could be saved through the use of fracture mechanics technology.

The theory of elasticity in its broad aspects, deals with a study of the behavior of those substances that possess the property of recovering their size and shape when the forces producing deformations are removed [2]. Also, as clearly indicated by the name itself, thermoelasticity is a branch of applied mechanics concerned with the effects of heat on the deformation and stresses in elastic bodies [2]. Although the theory of thermoelasticity has a long history, its foundations having been laid in the first half of the nineteenth century by Duhamel [3] and Neumann [4], widespread interest in this field did not develop until the years subsequent to World-War two.

The notion of elasticity was first introduced by Robert Hooke. He explained it in 1678 as "the power of any springy body is in the same proportion with the extension of that body" [2]. Cauchy [5] gave a formulation of the linear theory of elasticity, which is isothermal (under fixed temperature).

The theory of thermoelasticity deals with the effect of mechanical and thermal disturbances on an elastic body. In the nineteenth century, Duhamel [3] was the first to consider elastic problems with heat changes. In 1855 Neumann [4] re-derived the equations obtained by Duhamel. Their theory, the theory of uncoupled thermoelasticity, consists of the heat equation, which is independent of mechanical
effects, and the equation of motion, which contains the temperature as a known function. There are two defects of this theory. First, the fact that the mechanical state of the elastic body has no effect on the temperature which is not in accord with true physical experiments. Second, the heat equation being parabolic predicts an infinite speed of propagation for the temperature, which again contradicts physical observations.

In 1956, M. A. Biot [6] introduced the coupled theory of thermoelasticity. In this theory the equations of elasticity and of heat conduction are coupled which agree with physical experiments since any change of the temperature gives a certain amount of strain in an elastic body and vice versa. The theory of coupled thermoelasticity has proved useful for many problems. The equations of this theory consist of the equation of motion, which is a hyperbolic partial differential equation, and of the equation of energy conservation, which is parabolic. The nature of the second equation implies that if an elastic medium extending to infinity is subjected to a thermal or a mechanical disturbance, the effect will be felt instantaneously at infinity, which contradicts physical experiments. This shows the need for a new equation of energy, which is of hyperbolic type.

In 1967, Lord and Shulman [7] introduced the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body. This theory was extended in [8] by Sherief and in [9] by Dhaliwal and Sherief in 1980 to include the anisotropic case. In this theory a modified law of heat conduction
including both the heat flux and its time derivative replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both uncoupled and coupled theories of thermoelasticity.

This thesis consists of three chapters. The first chapter consists of six sections. Section 1 contains a review of the classical theory of elasticity. Section 2 contains a review of the essentials of the theory of thermodynamics such as the first and the second laws. Section 3 contains the derivation of the basic equations of the theory of uncoupled thermoelasticity showing its basic shortcomings. Section 4 contains the derivations of the basic equations of the coupled theory of thermoelasticity showing how this theory has eliminated one of the two shortcomings of the theory of uncoupled thermoelasticity and how this theory still predicts infinite speeds of propagation of heat waves contradictory to physical observations. Section 5 contains the derivations of the basic equations of the generalized theory of thermoelasticity with one relaxation time. It shows how this theory has dealt with the shortcomings of both the uncoupled and the coupled theories of thermoelasticity. In section 6, we introduce a brief review of crack problems in the theory of elasticity.

In chapter two we solve a dynamical problem for an infinite thermoelastic solid with an internal penny-shaped crack, which is subjected to a prescribed temperature and stress distributions. The problem is solved using both the Laplace and the Hankel transforms. The boundary conditions of the problem give a set of four
dual integral equations. These are reduced to two dual integral equations. The operators of fractional calculus are used to transform the dual integral equations into a Fredholm integral equation of the second kind, which is solved numerically. The inverse Hankel transform is performed exactly, while that of the Laplace transform is obtained using a numerical technique based on Fourier expansion methods. Numerical values for the temperature, stress and displacement are obtained and represented graphically. The stress intensity factor is also calculated and represented graphically.

In the third chapter we consider the problem of an infinite space with a linear finite mode I crack. The Sine and Cosine Fourier transforms as well as the Laplace transforms are used. The problem is reduced to the solution of a system of four dual integral equations. The solution of these equations is shown to be equivalent to the solution of a Fredholm integral equation of the First kind. This integral equation is solved numerically using the method of regularization. The inverse Laplace transforms are obtained numerically using a method based on the Fourier expansion techniques. Numerical values for the temperature, stress and displacement are obtained and represented graphically. The stress intensity factor is also calculated and represented graphically.